NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1563

EFFECT OF PARTIAL WING LIFT IN SEAPLANE LANDING IMPACT

By Stanley U. Benscoter

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

Washington
April 1948

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 1563

EFFECT OF PARTIAL WING LIFT IN SEAPLANE LANDING IMPACT

By Stanley U. Benscoter

SUMMARY

A solution is presented for the motion of a prismatic float of infinite length that is dropped vertically into the water at zero trim. A lift force, simulating seaplane wing lift, is assumed to act on the float and to remain constant during the impact period. The float is assumed to have a uniform mass per unit of length and a uniform wing lift per unit of length. The solution is determined for values of the lift ranging from zero to full wing lift. Time variations of the acceleration have been computed for a specific numerical example. The value of the mass ratio (ratio of apparent water mass to float mass) at the instant of maximum acceleration is determined in general form. The variation of maximum acceleration with sinking speed is illustrated for various amounts of wing lift.

INTRODUCTION

During a rough-water landing of a seaplane, some stalling of the wings may occur during the second and subsequent impacts. The wing lift becomes less than the weight of the seaplane. Solutions for the maximum acceleration in smooth water landings are available for both two- and three-dimensional cases of fluid flow when the wing lift equals the weight of the seaplane (references 1 and 2). These solutions may be converted to rough-water landings by assuming a reasonable shape for the wave surface. With partial wing lift the solution for the three-dimensional case becomes very complex mathematically. The effect of partial wing lift upon the maximum acceleration occurring during the impact is not known. However, the two-dimensional case can be readily solved. This solution offers valuable suggestions for the writing of structural design specifications.

SYMBOLS

a	acceleration
c	half loaded width
C	dimensionless parameter

g	acceleration due to gravity
H	momentum of fluid
k	wing lift factor (L/W)
L	wing lift per unit length
m	apparent water mass
M	float mass per unit length
t	time
W	weight of float per unit length
z	draft of float
β	dead-rise angle
ϵ	apparent-mass coefficient
η	dimensionless parameter
μ	mass ratio (m/M)

Subscripts:

o	instant of entry $(t = 0)$
m	instant of maximum acceleration

A dot is sometimes used to indicate differentiation with respect to time.

The magnitude of all physical quantities is assumed to be determined by a consistent system of units.

SOLUTION OF DIFFERENTIAL EQUATION

In the following analysis, a two-dimensional solution of a float entering water is made; that is, a prismatic float of infinite length is considered to enter the water at zero trim from a vertical drop. A lift force is assumed to be distributed over the length of the float in correspondence to the wing lift in a seaplane landing and is assumed to remain constant during the impact. The float is assumed to have a uniform mass per unit of length and a uniform wing lift per unit of length. A cross section of the float and coordinate system used is shown in figure 1. In the analysis, the equations are written for a unit slice of float and of fluid. The float has a mass M per unit

length. The fluid is assumed to have a total momentum that can be defined in terms of an apparent mass. In the present case, the apparent mass may be considered to be the mass of fluid contained in a half circular cylinder as originally introduced by von Kármán (reference 1) and shown in figure 1. The float is acted upon by a lift force L, its own weight W, and the water reaction. Forces and drafts are considered to be positive downward.

The apparent mass is proportional to the square of the loaded width. Consequently, with flat-side wedge-bottom floats, the apparent mass is also proportional to the square of the draft. The apparent mass and momentum of a unit slice of the fluid may be expressed in the following forms:

$$\left.\begin{aligned} m &= \epsilon z^2 \\ H &= m\dot{z} \end{aligned}\right\} \qquad (1)$$

In order to write an equation of motion governing the draft of the float, the float and the fluid may be considered to form a single system that is acted upon by the external forces W and L. Newton's law on the rate of change of momentum may then be written for the system as follows:

$$\frac{d}{dt}\left(M\dot{z} + m\dot{z}\right) = W - L \qquad (2)$$

This equation could be developed by a somewhat different argument. The second term $\frac{d}{dt}(m\dot{z})$ represents the rate of change of momentum of the fluid and thus may be regarded as the force exerted on the fluid by the float. If this quantity were preceded by a negative sign it would become the force exerted by the fluid on the float and would be written on the right-hand side of the equation. The right-hand side would then represent all of the forces acting on the float. The equation would then be regarded as an application of Newton's law of motion to the float itself.

A fairly large amount of experimental evidence is available to justify the use of this ideal fluid theory. Test results obtained in the NACA impact basin have shown good agreement with theory. The problem of the three-dimensional case for full wing lift has been solved and compared with experiment in reference 2.

The initial conditions for the solution of the differential equation are as follows: when $t = 0$,

$$\left.\begin{aligned} z &= 0 \\ \dot{z} &= \dot{z}_o \\ \ddot{z}_o &= a_o \end{aligned}\right\} \tag{3}$$

The initial velocity and initial acceleration are assumed to be known. The lift may be expressed by the product kW where k is considered herein to vary from 0 to 1. Introducing this value into equation (2) and performing the indicated differentiation gives

$$(M + m)\ddot{z} + \dot{z}\frac{dm}{dt} = (1 - k)W \tag{4}$$

This equation is nonlinear but may be readily converted into an integrable form. It is convenient in developing the solution of this equation to introduce the dimensionless mass ratio μ and the initial acceleration a_o:

$$\mu = \frac{m}{M} \tag{5}$$

$$a_o = \ddot{z}_o = (1 - k)g \tag{6}$$

Dividing through equation (4) by M and inserting the formulas (5) and (6) gives

$$(1 + \mu)\frac{d\dot{z}}{dt} + \dot{z}\frac{d\mu}{dt} = a_o \tag{7}$$

This equation may be integrated to obtain the following formula for velocity:

$$\dot{z} = \frac{\dot{z}_o + a_o t}{1 + \mu} \tag{8}$$

In this equation the appropriate value of the constant of integration has been introduced.

The equation for velocity may again be integrated to obtain the relationship between draft and time. Equation (8) may be written in the following form:

$$(1 + \mu)\, dz = \left(\dot{z}_o + a_o t\right) dt \tag{9}$$

After substitution of equations (1) and (5), equation (9) may be integrated to give the following formula:

$$z\left(1 + \frac{\mu}{3}\right) = \dot{z}_o t + \frac{a_o t^2}{2} \tag{10}$$

In this case the constant of integration is zero. This equation is a cubic equation in z or a quadratic equation in t. Hence, it is more convenient to solve for t in terms of the draft. Therefore,

$$t = -\frac{\dot{z}_o}{a_o} + \sqrt{\frac{\dot{z}_o^{\,2}}{a_o^{\,2}} + \frac{2}{a_o}\left(1 + \frac{\mu}{3}\right) z} \tag{11}$$

In the design of a seaplane the quantity that is required is the value of the acceleration. It may be noted that the original equation of motion is linear in the acceleration. Consequently, it may be solved to obtain a formula for acceleration in terms of drafts and velocities. Equation (7) may be solved for the acceleration to obtain

$$\ddot{z} = \frac{a_o}{1 + \mu} - \frac{\dot{z}}{1 + \mu}\frac{d\mu}{dt}$$

$$= \frac{a_o}{1 + \mu} - \frac{\dot{z}^2}{1 + \mu}\frac{d\mu}{dz} \tag{12}$$

From equations (1) and (5) it is seen that μ is related to the draft by the formula

$$\mu = \frac{\epsilon z^2}{M} \tag{13}$$

Differentiating equation (13) gives

$$\frac{d\mu}{dz} = \frac{2\epsilon z}{M} = \frac{2\mu}{z} \tag{14}$$

Substituting equations (8) and (14) into equation (12) gives the following formula for acceleration:

$$\ddot{z} = \frac{a_o}{1 + \mu} - \frac{2\mu}{z} \frac{(\dot{z}_o + a_o t)^2}{(1 + \mu)^3} \tag{15}$$

Equation (15) may be regarded as a formula for the acceleration in terms of time and draft. It is convenient to eliminate t in order to obtain a formula for acceleration in terms of the draft.

Equation (11) may be rearranged by transposing the first term on the right-hand side and squaring both sides.

$$\left(\dot{z}_o + a_o t\right)^2 = \dot{z}_o^{\,2} + 2a_o z \left(1 + \frac{\mu}{3}\right) \tag{16}$$

This formula may be substituted into equation (15) to obtain

$$\ddot{z} = \frac{-2\mu\dot{z}_o^{\,2}}{z(1 + \mu)^3} + \frac{a_o\left(3 - 6\mu - \mu^2\right)}{3(1 + \mu)^3} \tag{17}$$

If various values of z are assumed, the corresponding values of $\ddot{z}$ can be computed, use being made of equation (13) to determine μ. The associated values of t may be computed from equation (11). A graphical relation between acceleration and time may then be plotted. By use of

formula (17) for acceleration, a formula may be readily derived for the hydrodynamic reaction force. The resulting force formula would be in agreement with that for the case of no wing lift $(a_o = g)$ given in reference 3.

NUMERICAL EXAMPLE

For plotting purposes, it is convenient to express the acceleration in dimensionless form. Dividing through equation (17) by g gives

$$\frac{\ddot{z}}{g} = \frac{-2\mu}{(1 + \mu)^3}\left(\frac{\dot{z}_o^{\,2}}{gz}\right) + \frac{(1 - k)(3 - 6\mu - \mu^2)}{3(1 + \mu)^3} \tag{18}$$

In order to illustrate the variation of acceleration with time a simple example has been computed. The following values have been assumed for the float properties and the initial conditions:

Mass of float, M, slugs per foot	100
Apparent mass coefficient, ϵ, slugs per cubic foot	20
Initial velocity, $\dot{z}_o$, feet per second	12
Wing lift factor, k	1 or 2/3

The value $k = 1$ is for full wing lift; whereas, the value $k = \frac{2}{3}$ is a reasonable reduction factor for wing lift to be used in design practice. A graphical representation of the solution for the two cases is shown in figure 2. It may be seen that there is some reduction in the maximum acceleration with partial wing lift. However, the effect is small enough to be disregarded in most practical design cases.

MASS RATIO AT MAXIMUM ACCELERATION

In design practice it is actually necessary to know only the value of maximum acceleration that occurs during the impact. This maximum value may be found by differentiating the formula for acceleration with respect to time or draft. This derivative is set equal to zero. Thus,

$$\frac{d\ddot{z}}{dz} = 0 \tag{19}$$

Differentiation of equation (17) gives, after some rearrangement of terms,

$$a_o z_m\left(15 - 10\mu_m - \mu_m^{\,2}\right) - 3\left(5\mu_m - 1\right)\dot{z}_o^{\,2} = 0 \tag{20}$$

The subscript m has been introduced to indicate that the values of the quantities are those that occur at the instant of maximum acceleration.

Equation (13) shows that the draft may be expressed in terms of μ. Thus,

$$z = \sqrt{\frac{\mu M}{\epsilon}} \tag{21}$$

This formula holds true at all instants, including the instant of maximum acceleration. Hence, it may be used to eliminate the factor z_m in equation (20) by direct substitution. The result may be written in the following form:

$$\frac{\left(15 - 10\mu_m - \mu_m^{\,2}\right)\sqrt{\mu_m}}{3(5\mu_m - 1)} = \frac{\dot{z}_o^{\,2}}{a_o}\sqrt{\frac{\epsilon}{M}} = C \tag{22}$$

The parameter C depends upon the initial conditions and the float properties. Instead of solving for μ_m it is more convenient to assume various values of μ_m and compute the corresponding values of C. Such values are shown in the following table and are plotted in figure 3.

μ_m	C
0.20	∞
.22	19.94
.24	10.24
.26	6.99
.28	5.35
.30	4.28
.40	2.29
.50	1.53
.60	1.12
.80	.63
1.00	.33
1.20	.11
1.32	0

Equation (22) shows that for full wing lift $(a_o = 0)$, μ_m must have the value 0.2. This universal value of μ_m was originally given in reference 3.

MAXIMUM ACCELERATION

A formula for the relative acceleration at any instant is given by equation (18). The value of μ_m as determined from equation (22) may be substituted into equation (18) to determine the maximum acceleration. However, it is possible to develop a simplified formula for the acceleration which is applicable only at the instant of maximum acceleration.

Equation (20) may be rewritten by use of equation (6), in the following form:

$$\frac{\dot{z}_o^2}{gz_m} = \frac{(1 - k)\left(15 - 10\mu_m - \mu_m^2\right)}{3(5\mu_m - 1)} \tag{23}$$

Substitution of this equation into equation (18) gives, after combining terms,

$$\frac{\ddot{z}_m}{g} = -\frac{1 - k}{5\mu_m - 1} \tag{24}$$

In order to show the variation of acceleration with sinking speed speed $\dot{z}_o$, the parameter C (equation (22)) may be expressed as

$$C = \eta\left(\frac{1}{1 - k}\right) \tag{25}$$

where

$$\eta = \frac{\dot{z}_o^2}{g}\sqrt{\frac{\epsilon}{M}} \tag{26}$$

For a particular value of k, the parameter C may be determined for various assumed values of η from equation (25). The corresponding values of μ_m may be determined from equation (22) and substituted into

equation (24) to obtain accelerations. These accelerations may then be plotted against the parameter η. Such graphs, for various values of k, are shown in figure 4.

At full wing lift, equation (24) becomes indeterminate. The values $k = 1$ and $\mu_m = 0.2$ may be substituted into equation (18). Introducing equations (21) and (26) gives the formula

$$\frac{\ddot{z}_m}{g} = \frac{-25}{108\sqrt{0.2}}\eta = -0.518\eta \tag{27}$$

For this case the graph in figure 4 is a straight line.

CONCLUSIONS

The vertical drop of a prismatic float of infinite length with constant partial wing lift has been analyzed to determine the maximum acceleration. The float is assumed to have a uniform mass per unit of length and a uniform wing lift per unit of length. The mass ratio at the instant of maximum acceleration is found to depend upon a single dimensionless parameter involving the initial conditions and the float properties. The solution shows that partial wing lift may have a small effect upon the maximum acceleration. The effect is small enough, however, to be disregarded in most practical design cases.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., February 5, 1948

REFERENCES

1. von Kármán, Th.: The Impact on Seaplane Floats during Landing. NACA TN No. 321, 1929.

2. Mayo, Wilbur L.: Analysis and Modification of Theory for Impact of Seaplanes on Water. NACA TN No. 1008, 1945.

3. Sydow, J.: The Effect of Spring Support and Keeling on Landing Impact. (Über den Einfluss von Federung und Kielung auf den Landestoss.) British Air Ministry Translation No. 861. (From Jahrb. 1938 der deutschen Luftfahrtforschung, R. Oldenbourg (Munich), pp. I 329 - I 338.)

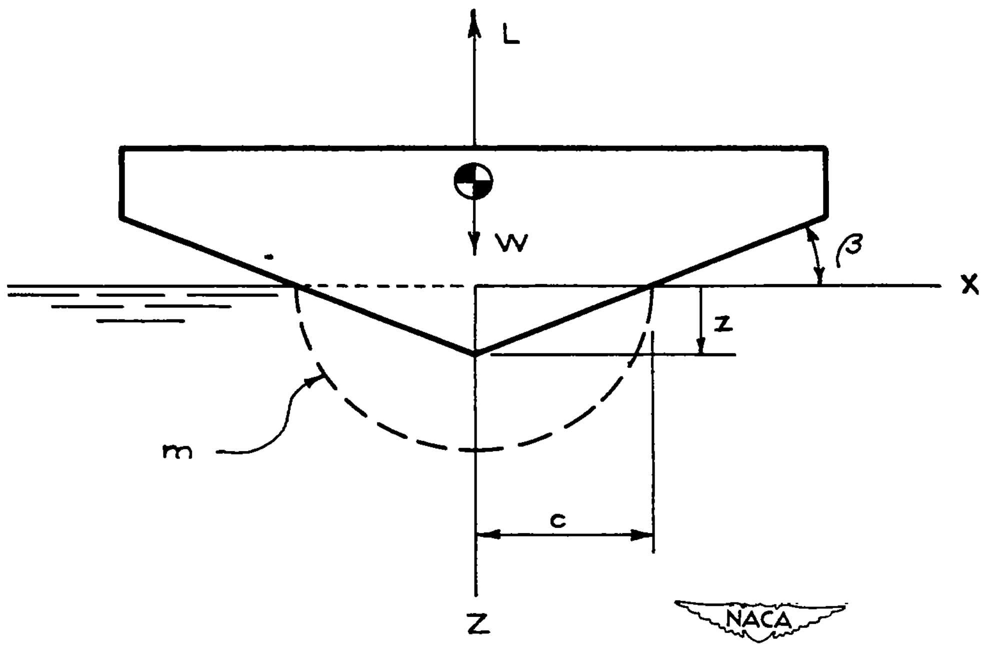

Figure 1.- Two-dimensional float entering fluid.

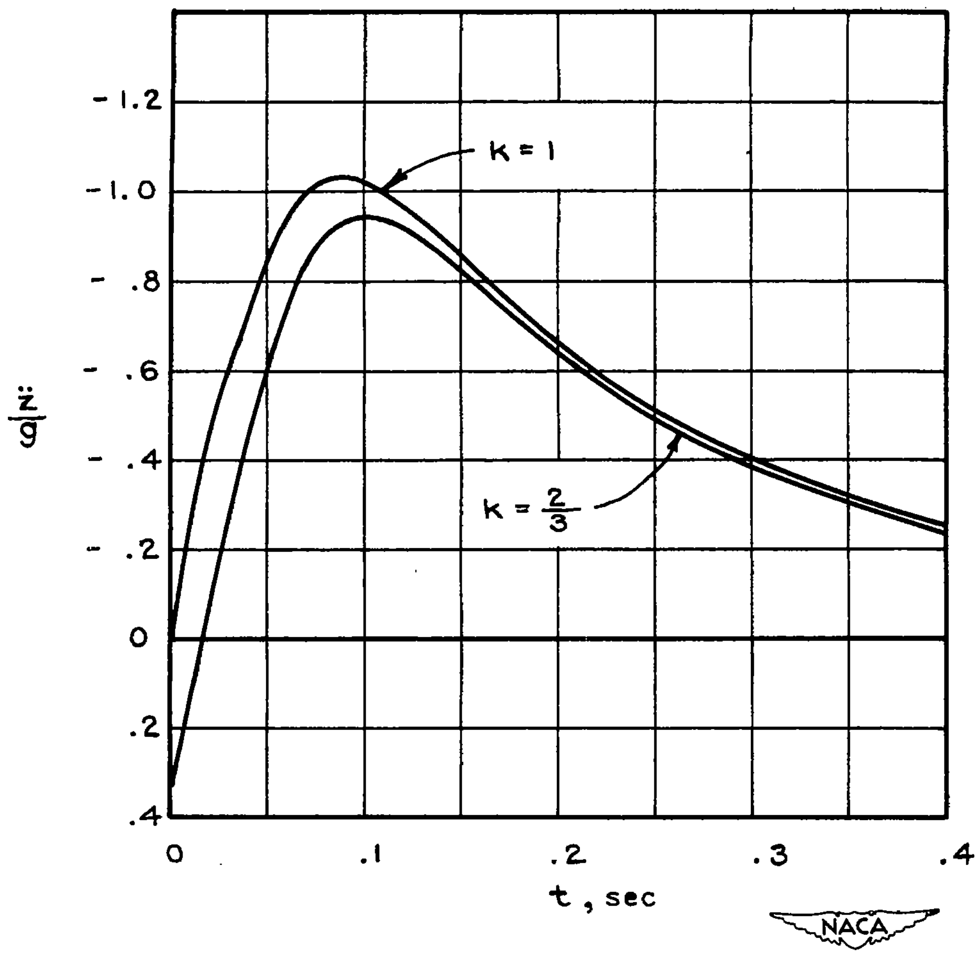

Figure 2.- Accelerations with full and partial wing lift.

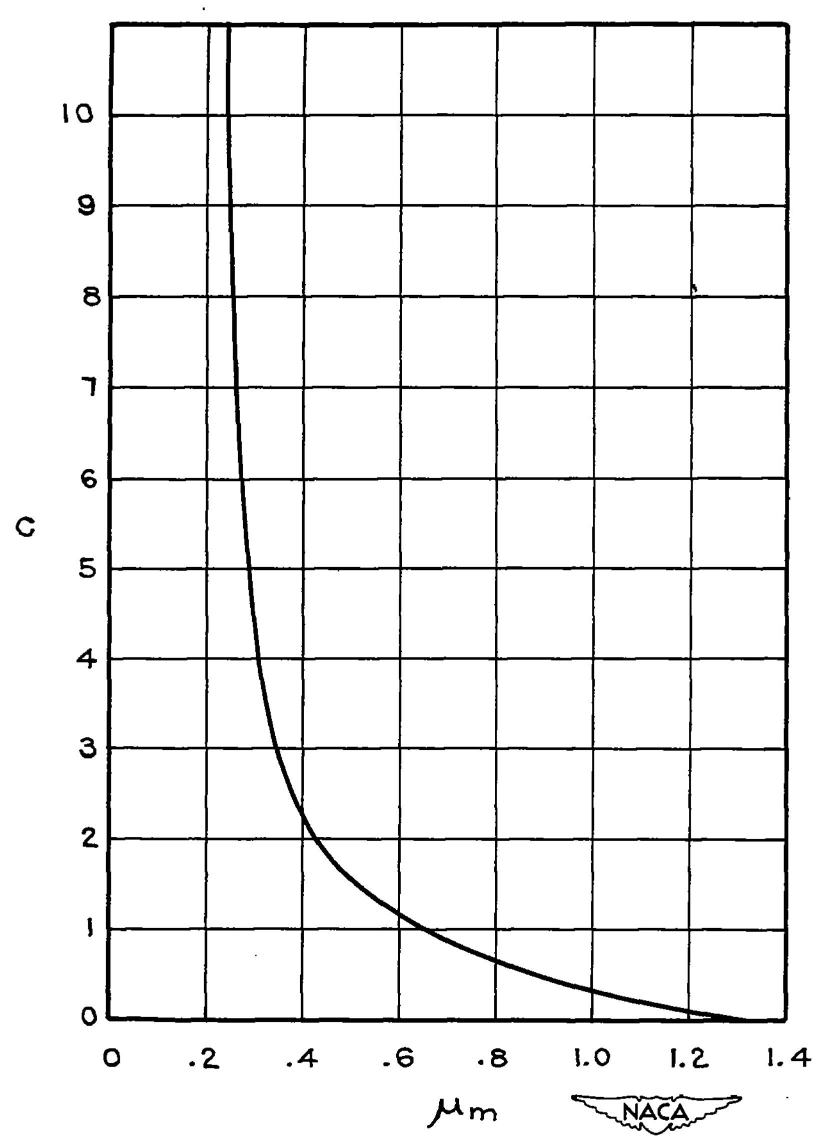

Figure 3.- Mass ratio at maximum acceleration.

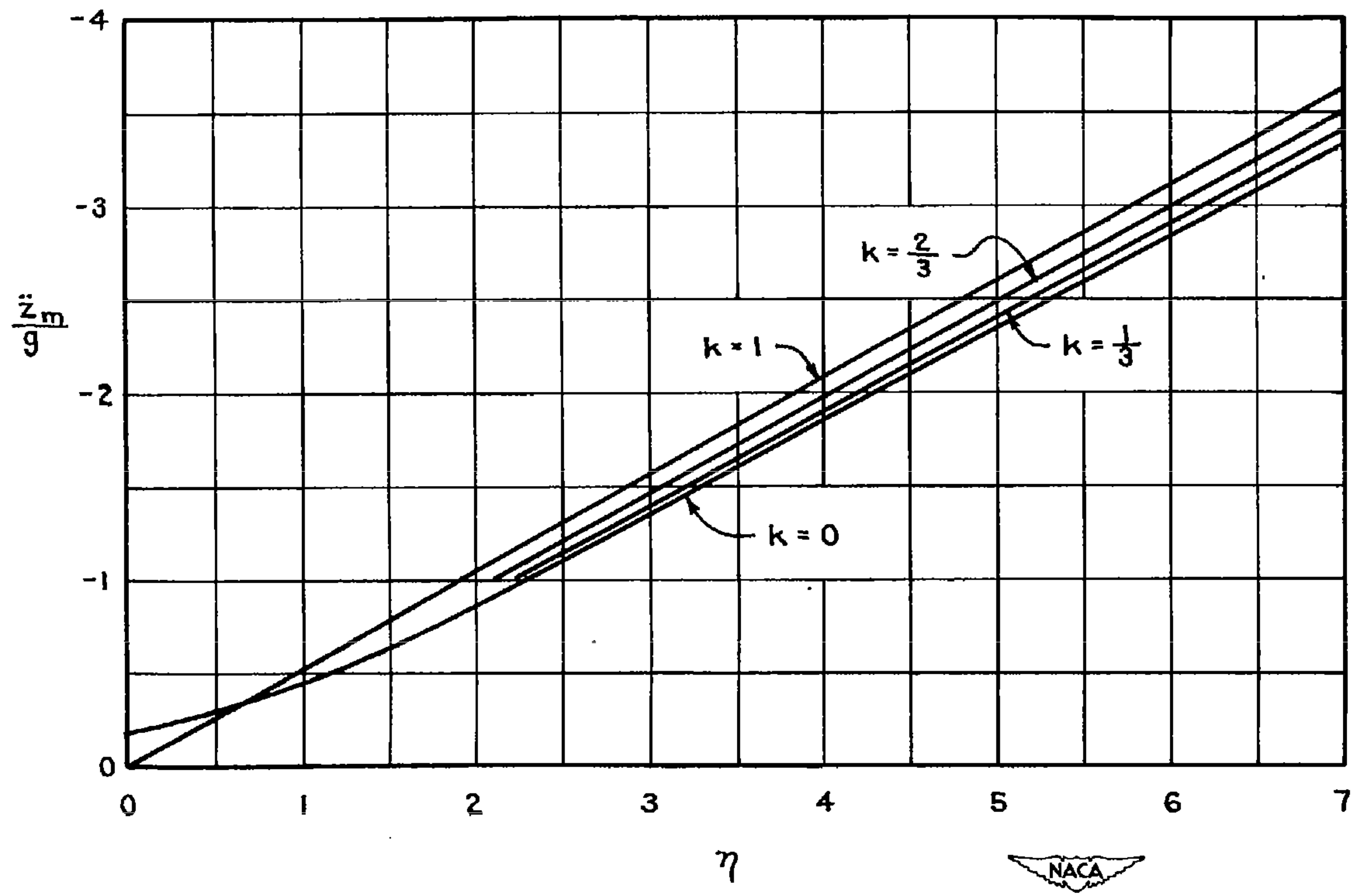

Figure 4.- Variation of acceleration with sinking-speed parameter η.